

# Surface dark solitons in nonlocal nonlinear media

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We predict the existence of surface dark solitons at the interface between a self-defocusing nonlocal nonlinear medium and a linear medium. The fundamental and higher-order surface dark solitons can exist when the linear refractive index of the self-defocusing media is much larger than that of the linear media. The fundamental solitons are stable and the stabilities of higher-order solitons depend on both nonlocality degree and propagation constant.

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Optical surface waves, a special type of waves that is localized at the interface between two different media, are widely used in sensing physical, chemical and biological agents to explore intrinsic and extrinsic properties of material surface. In the presence of nonlinearity, some kinds of surface solitons have been found theoretically and experimentally, such as in local Kerr media [1, 2], waveguide arrays [3, 4], photorefractive media [5, 6], and metamaterials [7]. Surface dark solitons existing at the interface between the local nonlinear media and the linear media have been studied [8, 9].

Nonlinear surface solitons can exist not only at the interface in local nonlinear media but also in nonlocal nonlinear media. Various types of surface solitons in nonlocal nonlinear media have been found and studied, such as bright surface fundamental solitons [10], incoherent surface solitons [11], surface dipoles [12, 13] and surface vortices [12]. In self-defocusing nonlocal media, the bright surface solitons and ring surface solitons have been predicted and studied [14, 15]. However, can surface dark solitons exist in self-defocusing nonlocal media? There is no study till now, though dark solitons in nonlocal self-defocusing medium were studied theoretically [16–18] and experimentally [19–22].

In this Letter, we address the existence of surface dark solitons (SDSs) at the interface between a self-defocusing thermal medium and a linear medium. We find that a SDS in a self-defocusing nonlocal media is identical with the half part of a dark soliton in the same bulk media when the linear refractive index of the self-defocusing media is much larger than that of the linear media. The similar relation between bright surface solitons and bulk solitons in nonlocal nonlinear media is firstly given in Ref. [23] under the same assumption.

We start our analysis by considering a TE polarized laser beam with a complex envelope  $q$  propagating along the  $z$  axis in the vicinity of the interface formed by a self-defocusing thermal nonlinear medium and a lin-

ear medium. The propagation of laser waves is described by the dimensionless (1+1)-D nonlocal nonlinear Schrödinger equation:

(i) in self-defocusing nonlinear media, i.e.  $x \leq 0$

$$i\frac{\partial q}{\partial z} + \frac{1}{2}\frac{\partial^2 q}{\partial x^2} + \Delta n q = 0, \quad (1a)$$

$$\Delta n - w_m^2 \frac{\partial^2 \Delta n}{\partial x^2} = -|q|^2, \quad (1b)$$

(ii) in linear media, i.e.  $x > 0$

$$i\frac{\partial q}{\partial z} + \frac{1}{2}\frac{\partial^2 q}{\partial x^2} - q n_d = 0. \quad (2)$$

Here the transverse  $x$  and longitudinal  $z$  coordinates are scaled in terms of beam width and diffraction length, respectively.  $\Delta n$  denotes the nonlinear perturbation of refractive index in the self-defocusing thermal medium.  $n_d > 0$  describes the difference of unperturbed refractive index between the self-defocusing thermal medium and the linear medium. The parameter  $w_m$  stands for the characteristic length of the nonlocal material response. We use  $w_m/w_0$  represents the degree of the nonlocality, where  $w_0$  is the beam width.

For the TE polarized wave, the continuity condition for transverse fields is  $q(x=+0) = q(x=-0)$ . Following the experimental instance [10], the continuity condition for the nonlinear refractive index is assumed as  $\partial \Delta n / \partial x|_{x=0} = 0$ , which can be achieved when the interface between nonlinear and linear media is thermally insulating. We assume  $n_d \gg 1$ , which is easily satisfied in an actual physical system [23]. When  $n_d \gg 1$  almost all the fields of laser beams will locate in the higher-index media [10–15, 23], so the boundary condition in the lower-index linear medium is  $q(x \rightarrow +\infty) = 0$ .

The asymptotic behaviors of SDSs when  $x \rightarrow -\infty$  are similar to that of bulk dark solitons [18]. If the stationary solution is in the form  $q(x, z) = W(x)e^{i\beta z}$ , we have  $W(x \rightarrow -\infty) = \sqrt{-\beta}$ . Therefore the propagation constant  $\beta$  must be negative. Consequently, one also have  $\Delta n(x \rightarrow -\infty) = \beta$ . In an actual thermal nonlinear system,  $\Delta n$  is proportional to the change of temperature

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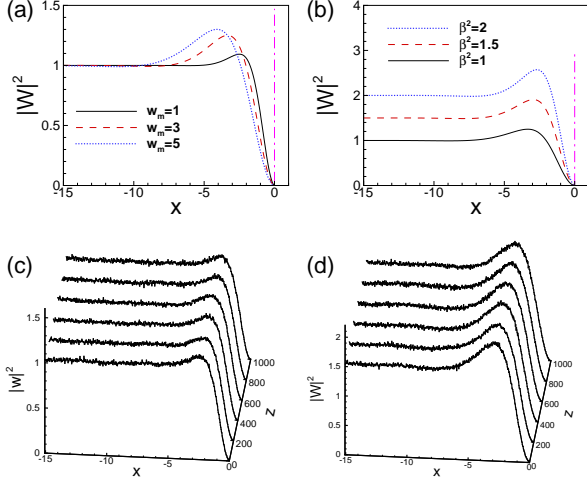


FIG. 1: (Color online) Profiles of fundamental SDSs for (a)  $w_m = 1, 3, 5$ ,  $\beta^2 = 1$ ; (b)  $w_m = 3$ ,  $\beta^2 = 1, 1.5, 2$ . Propagation of fundamental SDSs with (c)  $w_m = 1$ ,  $\beta^2 = 1$ ; (d)  $w_m = 3$ ,  $\beta^2 = 1.5$ . Purple dash-dotted lines show the interface.

due to the absorption of the wave energy. For a uniform distribution of optical intensity  $I = |q|^2 = |W|^2$  (i.e. in the vicinity of  $x \rightarrow -\infty$ ), the distribution of temperature could not be uniform because the temperature gradient is necessary for the transfer of heat energy. As a result, the exact uniform background for surface and bulk dark solitons in thermal nonlinear media can not be achieved. A much broad beam is used to approximate the uniform background (same in local case) in experiments[19, 20]. For the theoretical study of SDSs in this Letter,  $\partial\Delta n/\partial x|_{x \rightarrow -\infty} = 0$  and  $\partial q/\partial x|_{x \rightarrow -\infty} = 0$  are used in numerical simulations for the convenience.

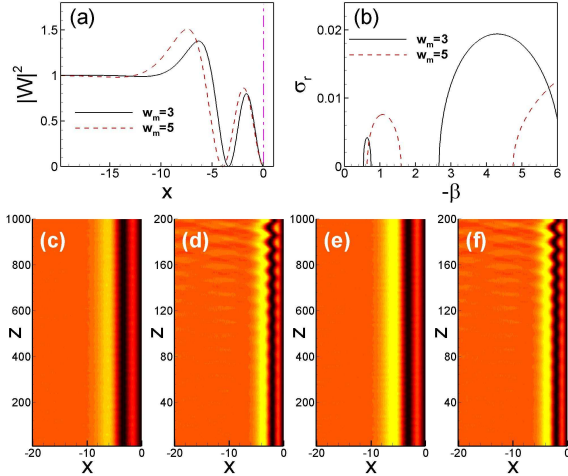


FIG. 2: (Color online) (a) Profiles of second-order SDS with  $w_m = 3, 5$ ,  $\beta^2 = 1$ . (b) The perturbation growth rate versus propagation constant for  $w_m = 3, 5$ . Propagation of second order SDS for (c)  $w_m = 3$ ,  $\beta^2 = 1$ ; (d)  $w_m = 3$ ,  $\beta^2 = 4.5$ ; (e)  $w_m = 5$ ,  $\beta^2 = 2.1$ ; (f)  $w_m = 5$ ,  $\beta^2 = 1$ .

We search for stationary solutions for Eqs.(1) and (2) numerically in the form  $q(x, z) = W(x)e^{i\beta z}$ , where  $W(x)$  is a real function and the propagation constant  $\beta < 0$ . The profiles of fundamental and higher-order SDSs are shown in Figs.1-4. We find the fields of SDSs reside all in the self-defocusing media with higher refractive index. The profiles of SDSs are proved to be identical with the half part of corresponding bulk dark solitons, which are obtained numerically from Eqs.(1) for a bulk medium ( $-\infty < x < \infty$ ).

To elucidate the linear stability of SDSs, we search for perturbed solutions in the form  $q(x, z) = [W(x) + a(x, z)]e^{i\beta z}$  and  $\Delta n = N(x) + b(x, z)$ , where  $|a(x, z)| \ll |W(x)|$  and  $|b(x, z)| \ll |N(x)|$  are real perturbations. Substituting the perturbed solutions into Eqs.(1) and (2), one can get the linearized equations around stationary solutions  $W(x)$  and  $N(x)$

(i) in nonlinear media,  $x < 0$ ,

$$i\frac{\partial a}{\partial z} + \frac{1}{2}\frac{\partial^2 a}{\partial x^2} - \beta a - (Na + Ub) = 0 \quad (3a)$$

$$b - w_m^2 \frac{\partial^2 b}{\partial x^2} = U^* a + U a^* \quad (3b)$$

(ii) in linear media,  $x > 0$

$$i\frac{\partial a}{\partial z} + \frac{1}{2}\frac{\partial^2 a}{\partial x^2} - (\beta + n_d)a = 0. \quad (4)$$

The perturbations  $a(x, z)$  and  $b(x, z)$  can grow with a complex rate  $\sigma$  upon propagation. Following the method in Ref.[24], one can get a linear eigenvalue problem for  $\sigma$ . The eigenvalue problem has been solved numerically and the instability growth rate  $\sigma_r$  (real part of  $\sigma$ ) are shown in Figs.2 -4. In addition, to confirm the results of linear stability analysis, we perform numerical simulations base on Eqs.(1) and (2). The solutions of SDSs obtained numerical are used as the incident profiles with noise in the form  $q(x, z = 0) = W(x)[1 + \rho(x)]$ , where  $\rho(x)$  is a random function with a Gaussian distribution and variance  $\sigma_{noise}^2 = 0.01$ .

Figure 1 shows results for the fundamental SDSs with different nonlocality degrees and different propagation constants. As the nonlocality degree increases, the shoulder of SDSs increases and moves away from the interface. The width of the intensity valley increases too. With increasing of  $\beta$ , the intensity of dark background increase and the width of valley decreases. The linear stability analysis shows the fundamental SDSs are always stable. The numerical simulations with noise-added incident profiles, shown in Figs.1(c) and (d), confirm the stability of SDSs.

Figure 2 shows results for the second-order SDSs with different nonlocality degrees. There are two valleys in the second-order SDSs as shown in Fig.2(a). As the nonlocality degree increases, the shoulder of the second-order SDSs increases and moves away from the interface. The width of two valleys increases together. The results of the linear stability analysis are shown in Fig.2(b). One

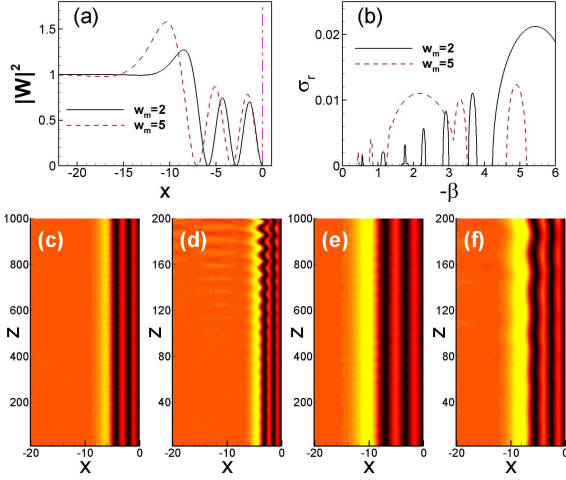


FIG. 3: (Color online)(a) Profiles of third-order SDS with  $w_m = 2, 5$ ,  $\beta^2 = 1$ . (b) The perturbation growth rate versus propagation constant for  $w_m = 2, 5$ . Propagation of third-order SDS for (c)  $w_m = 2$ ,  $\beta^2 = 2$ ; (d)  $w_m = 2$ ,  $\beta^2 = 4.6$ ; (e)  $w_m = 5$ ,  $\beta^2 = 1$ ; (f)  $w_m = 5$ ,  $\beta^2 = 2$ .

can see that the stability of the second-order SDSs depends on both  $w_m$  and  $\beta$ . The numerical simulations with noise-added incident profiles are shown in Figs.2(c) - (f). In Figs.2(c) and (e), the propagation are stable over 1000 times diffraction length, as predicated by the stability analysis[see Fig.2(b)]. In Figs.2(d) and (f) which are predicated unstable, the waves begin to oscillate over 100 times diffraction length..

Figure 3 shows results for the third-order SDSs with different nonlocality degrees. There are three valleys in the third-order SDSs as shown in Fig.3(a). As the nonlocality degree increases, the shoulder of the third-order SDSs increases and moves away from the interface. The width of three valleys increases together. The results of the linear stability analysis are shown in Fig.3(b). One can see that the stability of the third-order SDSs depends on both  $w_m$  and  $\beta$ . Comparing with the fundamental and second-order SDSs, the stable region for third-order are small. The numerical simulations with noise-added incident profiles are shown in Figs.3(c) - (f). In Figs.3(c) and (e), the propagation are stable over 1000 times diffraction length, as predicated by the stability analysis[see Fig.3(b)]. In Figs.3(d) and (f) which are predicated unstable, the waves begin to oscillate over 100 times diffraction length..

Figure 4 shows results for the forth-order SDSs with different nonlocality degrees. There are four valleys in the forth-order SDSs as shown in Fig.4(a). As the nonlocality degree increases, the shoulder of the forth-order SDSs

increases and moves away from the interface. The width of three valleys increases together. The results of the linear stability analysis are shown in Fig.4(b). One can see that the stability of the forth-order SDSs depends on both  $w_m$  and  $\beta$ . Comparing with the fundamental and

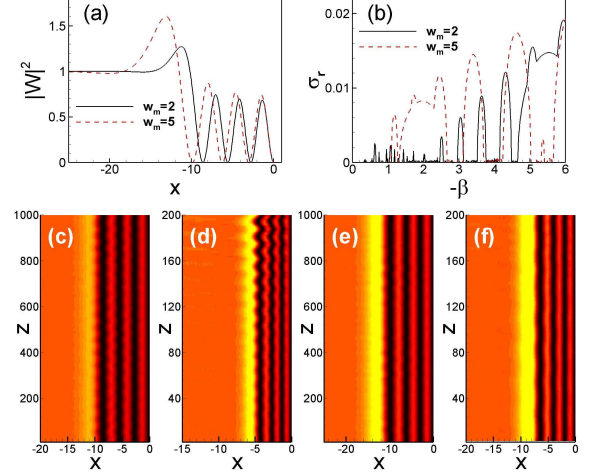


FIG. 4: (Color online)(a) Profiles of forth-order SDSs for  $w_m = 2, 5$ ,  $\beta^2 = 1$ . (b) The perturbation growth rate versus propagation constant for  $w_m = 2, 5$ . Propagation of the forth-order SDSs for (c)  $w_m = 2$ ,  $\beta^2 = 1$ ; (d)  $w_m = 2$ ,  $\beta^2 = 5$ ; (e)  $w_m = 5$ ,  $\beta^2 = 1$ ; (f)  $w_m = 5$ ,  $\beta^2 = 3.4$ .

second-order SDSs, the stable region for forth-order are much small. **We have noticed that there are some noise in the curves in Fig.4(b), the correctness of this results are still under examination.** The numerical simulations with noise-added incident profiles are shown in Figs.4(c) - (f). In Figs.4(c) and (e), the propagation are stable over 1000 times diffraction length. In Figs.4(d) and (f) which are predicated unstable, the waves begin to oscillate over 100 times diffraction length..

In summary, we introduced surface image dark solitons supported by the interface between self-defocusing thermal media and linear media. Such solitons integrate the unique features that are typical for surface waves supported by nonlocal nonlinear interfaces with those exhibited by dark solitons existing in nonlocal nonlinear media. Results motivate further study to nonlocal dark solitons and surface solitons.

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